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CHAOS THEORY AND ITS APPLICATIONS IN OUR REAL LIFE

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Abstract

Chaos theory is a mathematical field of study which states that non-linear dynamical systems that are seemingly random are actually deterministic from much simpler equations. Chaos theory was developed by inputs of various mathematicians and scientists; its applications are found in a large number of scientific fields. The purpose of this paper is to provide an introduction to chaos theory together with fractals, the elaborate patterns which have become its emblem. This paper we discuss chaotic systems, Fractals and its application, real life application of chaos theory and limitations of chaos theory. Finally we establish the idea of control of chaos.

***Keywords*:** Chaos theory, Fractals, Sensitive dependence on initial conditions (SDIC)

Introduction

The word Chaos comes from the Greek word "Khaos", meaning "gaping void". Mathematicians say it is tough to define chaos, but is easy to "recognize it when you see it." Chaos in other words means a state of total confusion or predictability in the behavior of a complex natural system. Chaos theory (Devaney 1989) is the concept that a small change now can result in a very large change later. It is a field of study in mathematics, with applications in several disciplines including physics, engineering, economics, biology (Morse 1967), and philosophy which primarily states that small differences in initial conditions (such as those due to rounding errors in numerical computation) can yield widely diverging outcomes for chaotic systems, rendering long-term prediction impossible in general. I hope that this paper serves as a useful tool for anyone who is interested in understanding this topic.

Chaos theory is one of the fundamental theories in our lives. It is the study of complex, nonlinear dynamic systems. It is a branch of mathematics that deals with systems that

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appear to be orderly (deterministic) but, in fact, harbor chaotic behaviors. It also deals with systems that appear to be chaotic, but, in fact, have underlying order. In other words, the deterministic nature (Robert 1976) of these systems does not make them predictable. This behavior is known as deterministic chaos, or simply chaos. Nature is highly complex, and the only prediction you can make is that she is unpredictable. Chaos Theory has managed to somewhat capture the beauty of the unpredictable and display it in the most awesome patterns. Nature, when looked upon with the right kind of eyes, presents her as one of the most fabulous works of art ever. Chaos Theory (Lorenz 1963) holds to the axiom that reality itself subsists in a state of ontological anarchy.

The phenomenon of Chaos theory was introduced to the modern world by Edward Lorenz in 1972 with conceptualization of 'Butterfly Effect'. Understanding this theory will help make a complex system more predictable. Thus when working with a system you should be aware of all the inputs and keep them controlled. As chaos theory was developed by inputs of various mathematicians and scientists, its applications are found in a large number of scientific fields. Lorenz was a meteorologist who developed a mathematical model used to model the way the air moves in the atmosphere. It caused vast differences in the outcome of the model. In this way he discovered the principle of Sensitive Dependence on Initial Conditions (SDIC), which is now viewed as a key component in any chaotic system. A multidisciplinary interest in chaos, complexity and self-organizing systems started in 1970's with the invention of computers. Benoît Mandlebrot found the piece of the chaos puzzle that put all things together. Mandelbrot published a book, The Fractal Geometry of Nature (Devaney and Keen 1989), which looked into a mathematical basis of pattern formation in nature, much like the earlier work of Turing. His fractals (the geometry of fractional dimensions) helped describe or picture the actions of chaos, rather than explain it. Chaos and its workings could now be seen in color on a computer.

In the early 1970's, May was working on a model that addressed how insect birthrate varied with food supply. He found that at certain critical values, his equation required twice time to return to its original state- the period having doubled in value. After several period-doubling cycles, his model became unpredictable, rather like actual insect populations tend to be unpredictable. Since May's discovery with insects, mathematicians have found that this period-doubling is a natural route to chaos for many different systems.

The deterministic chaos implies the uniqueness and distinguishable evolution of each individual trajectory in the system. That is why it is unpredictable, i.e. a single trajectory cannot be completely predictable for all future or past times, unless all the initial data of that individual trajectory is exactly known. But, if the number of individual trajectories is too large, or infinite, the probability to know exactly the initial data of one of them, is usually equal to zero.

Chaotic Systems

Chaotic systems are unstable since they tend not to resist any outside disturbances but instead react in significant ways. In other words, they do not shrug off external influences but are partly navigated by them. These systems are deterministic because they are made up of few, simple differential equations, and make no references to implicit chance mechanisms. A deterministic system is a system in which no randomness is involved in the development of future states of the system. It is said to be chaotic whenever its evolution depends on the initial conditions. This property implies that two trajectories emerging from two different close by initial conditions. However, only in the last thirty years of twentieth century, experimental observations have pointed out that. In fact, chaotic systems are common in nature. Many natural phenomena can also be characterized as being chaotic. They can be found in meteorology, solar system, heart and brain of living organisms and so on.

Characteristics of a chaotic system:

(i) No periodic behavior.

(ii) Sensitivity to initial conditions.

(iii) Chaotic motion is difficult or impossible to forecast.

(iv) The motion looks random.

(v) Non-linear.

Because of the various factors involved in chaotic systems, they are hard to predict. A lot of complicated and computations and mathematical equations are involved. Solutions of chaotic systems can be complex and typically they cannot be easily extrapolated from current trends. The game of Roulette is an interesting example that might illustrate the distinction between random and chaotic systems. If we study the statistics of the outcome of repeated games, then we can see that the sequence of numbers is completely random. Finally chaotic systems are very sensitive to the initial condition which means that a slight change in the starting point can lead to enormously different outcomes. This makes the system fairly unpredictable.

Fractals and its applications

Fractals are not just complex shapes and pretty pictures generated by computers. Anything that appears random and irregular can be a fractal. Fractals permeate our lives, appearing in places as tiny as the membrane of a cell and as majestic as the solar system.

Fractals are the unique, irregular patterns left behind by the unpredictable movements of the chaotic world at work.

In theory, one can argue that everything existent on this world is a fractal:

- the branching of tracheal tubes,
- the leaves in trees,
- the veins in a hand,
- water swirling and twisting out of a tap,
- a puffy cumulus cloud,
- tiny oxygene molecule, or the DNA molecule etc.

According to Kenneth Falconer (1985), a fractal exhibits the following properties:

(i) Ability to be differentiated and to have a fractal dimension.

(ii) Self-similarity (exact, quasi self-similarity, statistical or qualitative).

(iii) Multifractal scaling.

(iii) Fine and detailed structure at any scale.

(iv) Simple and perhaps recursive definitions.

Fractals have always been associated with the term chaos. One author elegantly describes fractals as "the patterns of chaos". Fractals depict chaotic behavior, yet if one looks closely enough, it is always possible to spot glimpses of self-similarity within a fractal (Devaney and Keen 1989).

To many chaologists, the study of chaos and fractals *(Falconer 2014)* is more than just a new field in science that unifies mathematics, theoretical physics, art, and computer science - it is a revolution. It is the discovery of a new geometry, one that describes the boundless universe we live in; one that is in constant motion, not as static images in textbooks. Today, many scientists are trying to find applications for fractal geometry, from predicting stock market prices to making new discoveries in theoretical physics.

Fractals have more and more applications in science. The main reason is that they very often describe the real world better than traditional mathematics and physics. Let us now examine the mathematical construction of a typical fractal *(Falconer 1997)* curve and the properties that it has. This fractal is called Koch's snowflake, because its shape resembles that of a snowflake and it was first conceived by Helge von Koch, a Swedish mathematician. It can be seen in the following Fig. 1. The algorithm for its construction is the following:

(i) From an equilateral triangle, remove the middle third of each side.

(ii) Draw another equilateral triangle, with its sides being equal to one third of the sides of the initial triangle, one of its sides replacing the line segment removed, and the other two sides lying outside the initial triangle.

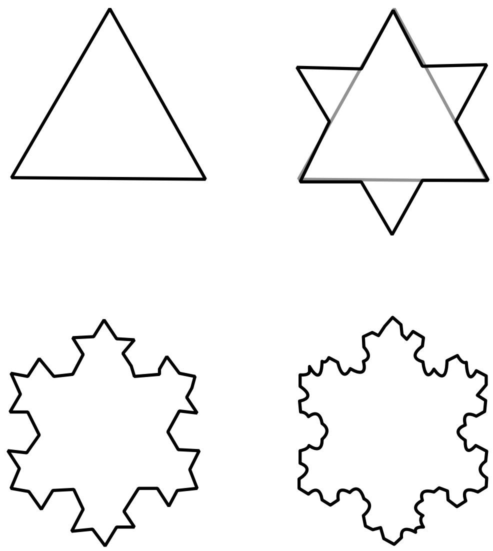

Fig. 1. The Koch Snowflake.

If this algorithm is executed ad infinitum, we can observe some very interesting properties this "snowflake" has. For example it displays exact self-similarity that is it is exactly the same as the initial curve no matter how much we zoom in.

The study of turbulence in flows is very adapted to fractals (Devaney and Keen 1989). Turbulent flows are chaotic and very difficult to model correctly. A fractal representation of them helps engineers and physicists to better understand complex flows. Flames can also be simulated. Porous media have a very complex geometry and are well represented by fractal. This is actually used in petroleum science. The flow of water in a river for example can seem very disorderly and difficult to monitor, but chaos theory and fractals can more accurately describe this kind of flow, and further advancements are expected in the future. Snowflakes, broccoli, coastlines and mountain ranges are some self-similar natural objects which can be described as fractals. Of course, the difference between mathematical fractals and natural fractals is that the self-similarity in the latter is not exact, as it is in the former, but it is quasi-self similarity, and that we cannot see a natural fractal at an infinitely small scale. The Barnsley Fern is an example of natural fractals.

Fig. 2. The Barnsley Fern.

Astronomy can be studied by using fractals. Fractals (Baker and Gollub 1990) will maybe revolutionize the way that the universe is seen. Cosmologists usually assume that matter is spread uniformly across space. But observation shows that this is not true. Astronomers agree with that assumption on "small" scales, but most of them think that the universe is smooth at very large scales. However, a dissident group of scientist's claims that the structure of the universe is fractal at all scales. If this new theory is proved to be correct, even the big bang models should be adapted. Some years ago we proposed a new approach for the analysis of galaxy and cluster correlations based on the concepts and methods of modern Statistical Physics. This led to the surprising result that galaxy correlations are fractal and not homogeneous up to the limits of the available catalogues. In the meantime many more red shifts have been measured and we have extended our methods also to the analysis of number counts and angular catalogues. The result is that galaxy structures are highly irregular and self-similar. The usual statistical methods, based on the assumption of homogeneity, are therefore inconsistent for all the length scales probed until now. A new, more general, conceptual framework is necessary to identify the real physical properties of these structures. But at present, cosmologists need more data about the matter distribution in the universe to prove (or not) that we are living in a fractal universe.

Fractals are used to describe the beauty of nature. For example we take a tree, Pick a particular branch and study it closely. Choose a bundle of leaves on that branch. To chaologists, all three of the objects described - the tree, the branch, and the leaves - are identical. To many, the word chaos suggests randomness, unpredictability and perhaps even messiness. Chaos is actually very organized and follows certain patterns. The problem arises in finding these elusive and intricate patterns. One purpose of studying chaos through fractals is to predict patterns in dynamical systems that on the surface seem unpredictable. A system is a set of things, an area of study, a set of equations is a system, as well as more tangible things such as cloud formations, the changing weather, the movement of water currents, or animal migration patterns. Weather is a favorite example for many people. Forecasts are never totally accurate and long-term forecasts, even for one week, can be totally wrong. This is due to minor disturbances in airflow, solar heating, etc. Each disturbance may be minor, but the change it creates will increase geometrically with time. The weather will be far different than what was expected. With fractal geometry we can visually model much of what we witness in nature, the most recognized being coastlines and mountains. Fractals are used to model soil erosion and to analyze seismic patterns as well.

A new application is fractal-shaped antennae that reduce greatly the size and the weight of the antennas. Fractenna is the company which sells these antennae. The benefits depend on the fractal applied, frequency of interest, and so on. In general the fractal part produces 'fractal loading' and makes the antenna smaller for a given frequency of use. Practical shrinkage of 2-4 times are realizable for acceptable performance. Surprisingly high performance is attained. Fractals are used to describe the roughness of surfaces. A roug surface is characterized by a combination of two different fractals. Biosensor interactions can be studied by using fractals (Devaney and Keen 1989).

Actually, the most useful use of fractals in computer science is the fractal (Devaney and Keen 1989)image compression. This kind of compression uses the fact that the real world is well described by fractal geometry. By this way, images are compressed much more than by usual ways (e.g. JPEG or GIF file formats). Another advantage of fractal compression is that when the picture is enlarged, there is no pixelisation. The picture seems very often better when its size is increased.

Applications of Chaos Theory

This section we discuss the current possible applications of chaotic systems in mathematics and another field of our real life. The applications of chaos have proven to be an exciting and fruitful. Chaos theory was born from observing weather patterns, but it has become applicable to a variety of other situations. Some areas benefiting from chaos theory today are mathematics, geology, microbiology, biology, computer science, economics, engineering, finance, algorithmic trading, meteorology, philosophy, anthropology, physics, politics, population dynamics, physiology, and robotics. Besides there are so many comprehensive list as new applications are appearing. These systems include weather models, the stock market, bird migration patterns, behavior of boiling water, neural networks and systems related to quantum phenomena. This theory is based on two main components; the first one is that systems, regardless of their degree of complexity, depend on an underlying overall equation or a principle that governs their behavior thus making it deterministic, theoretically, which is not due to its instability and the presence of a large number of contributing factors. The second main component is the high sensitivity to initial conditions, that a minute change in the initial conditions, such as rounding errors in numerical computation of a certain dynamical system can produce cataclysmic and unpredictable outcomes for that dynamical system. Now we describe the following applications of chaos theory in our real life.

(i) Chaos theory in Stock Market

Chaos analysis has determined that market prices are highly random, but with a trend. The amount of the trend varies from market to market and from time frame to time frame. The price movements that take place over the period of several minutes will resemble price movements that take place over the period of several years. In theory. big market crashes should never happen. But Mandlebrot predicts that a market crash should occur about once a decade. Given the fact that we have had major crashes in 1987,1998 and 2008-roughly once a decade-it's clear that Mandlebrot made a pretty good prediction.

The new Fractal Market Hypothesis, based on Chaos theory explains the phenomena in financial branch, which the Efficient Market Hypothesis could not deal with. In the hypothesis, Hurst exponent determines the rate of chaos and distinguished fractal from random time series. Lyapunov exponent determines the rate of predictability. A positive Lyapunov exponent indicates chaos and it sets the time scale which makes the state of prediction possible. Plotting stock market variations and matching them with chaotic analyses of above exponents, one might predict future behavior of market.

(ii) Chaos Theory in the Garment Industry and Fashion Design

In this paper, we use chaos theory to explain the phenomenon in the field of fashion design. First we have to do a comparative analysis between nonlinear and garment industry trends of chaos theory, and then discuss the effect of Butterfly Effect to the product positioning of clothing brand, study the relationship of the fractal theory between the integrity and locality of the garment industry brand design. Finally, make examples of the wide range of applications of chaos theory in the field of artistic form design. This paper introduces chaos theory into the field of garment industry and design, which will bring far-reaching influence to the development of the garment industry and the future design.

(iii) Chaos in the Human Body

Chaos theory can also be applied to human biological rhythms. The human body is governed by the rhythmical movements of many dynamical systems: the beating heart, the regular cycle of inhaling and exhaling air that makes up breathing, the circadian rhythm of waking and sleeping, the jumping movements of the eye that allow us to focus and process images in the visual field, the regularities and irregularities in the brain waves of mentally healthy and mentally impaired people as represented on electroencephalograms. None of these dynamic systems are perfect all the time and when a period of chaotic behavior occurs, it is not necessarily bad. Applying chaos theory to these human dynamic systems provides information about how to reduce sleep disorders, heart disease and mental disease.

It has been argued that some cardiac arrhythmias are instances of chaos. This opens the doors to new strategies of control. The traditional method of controlling a system is to model it mathematically in sufficient detail to be able to control critical parameters. However, this method fails in chaotic systems since no model can be developed for a system with an infinite number of unstable orbits. The OGY method mentioned above was able to exploit the properties of chaotic mechanical and electrical systems; however, system-wide parameters in the human body cannot be manipulated quickly enough to control cardiac chaos. Therefore, Garfinkel, Spano, Ditto and Weiss (1992) developed a similar method which they called proportional perturbation feedback (PPF).

In eleven separate experimental runs, the technique was successful at controlling induced arrhythmia in eight cases. The good thing is that the stimuli did not simply over drive the heart; stimuli did not even have to be delivered on every beat. This contrasted well with the periodic method which was never successful in restoring a periodic rhythm, and even showed a tendency to make the rhythm more a periodic. Therefore, besides providing a successful method of control (Richard, 1994), the method would be a less dramatic intrusion into the patient’s system.

Similar efforts are being made to control epileptic brain seizures which exhibit chaotic behavior. This technique, controls by waiting for the system to make a close approach to an unstable fixed point along the stable direction. It then makes a minimal intervention to bring the system back on the stable manifold (Robert 1976). Again, an important benefit is the minimal amount of intervention required to control the chaotic event.

Redington and Reidbord (1992) attempted to demonstrate that the human heart could display chaotic traits. They monitored the changes in between-heartbeat intervals for a single psychotherapy patient as she moved through periods of varying emotional intensity during a therapy session. Results were admittedly inconclusive. Not only were there ambiguities in the various plots the authors produced to purportedly show evidence of chaotic dynamics (spectral analysis, phase trajectory, and autocorrelation plots), but when they attempted to compute a Lyapunov exponent as more definitive confirmation of chaotic behavior, the authors found they could not reliably do so.

(iv) Chaos in the Social Sciences

Some researchers in the field of social sciences (Kiel and Elliott 1996) even propose that the chaos theory offers a revolutionary new paradigm, away from the materialistic Utopia, and that social system should be maintained *at the edge of chaos*, between too much and too little authoritarian controls. This comment concerns politics rather than physics. The application of chaos models in the analysis of social phenomena is accompanied by some important scientific problems. First, whether observations of social phenomena are generated by nonlinear dynamics cannot be ascertained beyond considerable doubt, especially when these observations contain a measurement error that is there is a problem of external validity. Secondly, and more important, as a theory of irregular cyclical social behavior is lacking, inductive-statistical theory formation about such behavior, which is based on fitting a mathematical model of chaos to observations of social phenomena, is impossible unless additional information is used concerning the context and circumstances wherein the social phenomena occur; that is the internal validity of any theoretical explanation that is derived from only a fitted mathematical model (of chaos) cannot be assessed. So research into the suggestion derived from mathematical chaos theory that irregular cycles may be present in the development of social phenomena over time requires theory-formation about irregular cyclical social behavior on the basis of established theoretical insights and empirical evidence instead of fitting sophisticated mathematical models of chaos to observations of social phenomena.

(v) Chaos in Engineering

Even though long-term prediction may fail if a system is chaotic, an engineer need not be over- concerned about this failure. Rarely does an engineer need to predict the future state of a system so accurately. An engineer is more concerned with the overall properties of the orbit of a system. Even if one doesn't know the future state of the system, from the numerical solution of the concerned differential equations one can say with great confidence that the state will not run to infinity, will not collapse, and the state will be "somewhere" within a definite volume of the state space. One of the utilities of chaos is that it can provide a framework for analyzing where on the spectrum between pure signal and pure noise, a data set might fall. Chaos is a type of signal, but can appear to be noise if not analyzed properly. Chaotic signals are irregular in time, but highly structured in phase space. Phase space embedding therefore provides a tool for visualizing the structure of chaotic signals, and for distinguishing chaos from noise. Furthermore, noise, by definition, is infinitely dimensional, whereas chaos is (relatively small) finite dimensional. Time series data can therefore be "unfolded" into higher dimensional space by sampling data points at fixed distances. A new data point will be created from a single time point and some integer number of steps ahead of that time point.

(vi) Robotics

This paper presents a summary of applications of chaos and fractals in robotics. Firstly, basic concepts of deterministic chaos and fractals are discussed. Then, fundamental tools of chaos theory used for identifying and quantifying chaotic dynamics will be shared. Principal applications of chaos and fractal structures in robotics research, such as chaotic mobile robots, chaotic behavior exhibited by mobile robots interacting with the environment, chaotic optimization algorithms, chaotic dynamics in bipedal locomotion and fractal mechanisms in modular robots will be presented. A brief survey is reported and an analysis of the reviewed publications is also presented.

Besides the Lyapunov exponent and fractal dimension, other quantifiers should be employed for analysis, which will broaden our understanding of robot dynamics. Until now, researchers have found several routes to chaos in the gait patterns of very simple passive dynamic bipeds. The analyses of complex biped models, which are adequately closer to the behavior of real biological systems, will provide deep insight into the origin of chaotic dynamics and bifurcation scenarios. Swarm intelligence (SI) has emerged as an interdisciplinary research area for scientists and engineers. Like Chaotic ABC, Chaotic PSO and Chaos ACO should be employed in motion planning of autonomous agents. The applications of chaotic dynamics will generate efficient motion planning techniques for mobile robots. A brief survey is reported and an analysis of the reviewed publications is also presented. The final aim of robotics is the creation of intelligent autonomous robots. The dynamical system theory is the right answer for a dynamic world. From this review of papers, it is evident that deterministic chaos is an overwhelming idea in science and an omnipresent phenomenon in various robotic domains. Physiological systems are inspiring the control systems and the physical shapes of the robots. Scientists and engineers are striving to realize the decades-old dream of a versatile, mobile, general-purpose autonomous robot. Chaos theory, combined with other important technologies such as artificial intelligence, machine learning and nonlinear optimal control, will help realize this goal in the offing.

(vii) Chaos in Circuits

The Chua circuit (Chen and Ueta, 2002) is among the simplest non-linear circuits that show most complex dynamical behavior, including chaos which exhibits a variety of bifurcation phenomena and attractors. In recent years chaos theory has attracted much interest in both the academic area and engineering study. One of the great achievements of the chaos theory is the application in secure communications. Chaotic signals depend very sensitively on initial conditions, have unpredictable features and noise like wideband spread spectrum. So, it can be used in various communication applications because of their features of masking and immunizing information against noise. The chaos communication fundament is the synchronization of two chaotic systems under suitable conditions if one of the systems is driven by the other. Chua's circuit is a simple oscillator circuit which exhibits a variety of bifurcations and chaos. The circuit (Chen and Ueta, 2002) contains three linear energy storage elements (an inductor and two capacitors), a linear resistor, and a single nonlinear resistor.

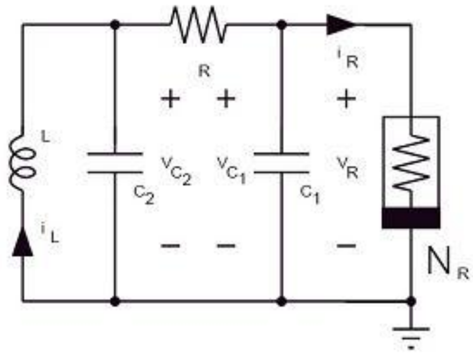

Fig. 3. Schematic of Chua’s Circuit. NR is the active nonlinear resistor, called as the Chua diode.

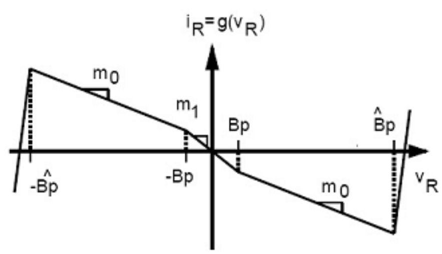

Fig. 4. Piece wise linear characteristic of the nonlinear resistor NR in Chua’s circuit.

(viii) Chaos Theory as Literary Theory

As a literary theory (Yasser, KRA. 2007) chaos theory helps readers more deeply understand and appreciate the complex ideas behind some works of literature we might encounter. For example, Shakespeare’s Hamlet, in many respects, perfectly illustrates many of the core principles of chaos theory. Hamlet himself, in fact, seems to possess a particular awareness of the chaotic nature of human existence. Throughout the play, Hamlet constantly questions not only his own motives and action and their possible ramifications and effects, but also those of the various forces both those that are natural and apparently supernatural that are conspiring around him. Hamlet itself highlights, in miniature, the various seemingly unpredictable and chaotic forces that control reality. In a sense, the play itself makes use of a version of the famous butterfly effect that would be postulated more than three centuries later: The death of Hamlet's father results, ultimately, in the utter collapse of the entire kingdom of Norway and the death of nearly every major character in the play. The entire world in which Hamlet lives – his entire reality, in fact both external and internal – is depicted as being radically shifted by the death of a single human being. The events that are depicted and examined in the play then, illustrate the chaotic, complex, and ultimately unpredictable and seemingly random and determined forces upon which reality is structured. Reading Shakespeare's Hamlet with a firm knowledge of chaos theory to reveal a surprising measure of awareness on Shakespeare's that predates the scholarly exploration of chaos theory by nearly four centuries.

(ix) Chaos to Produce Music

The goal is to inspire composes from the generated ideas. A chaotic mapping provides a technique for generating musical (Boon and Decroly 1995) variations of an original work. This technique, based on the sensitivity of chaotic trajectories to initial conditions, produces changes in the pitch sequence of a piece. A sequence of musical pitches p_i is paired with the *x*-components $\{x_i\}$ of a Lorenz chaotic trajectory. In this way, the *x* axis becomes a pitch axis configured according to the notes of the original composition. Then a second chaotic trajectory, whose initial condition differs from the first, is launched. Its *x*-components trigger pitches on the pitch axis (via the mapping) that vary in sequence from the original work, thus creating a variation. There are virtually an unlimited number of variations possible, many appealing to experts and other alike.

The technique's success with a highly context-dependent application such as music, indicates it may prove applicable to other sequences of context dependent symbols. e.g. DNA or protein sequences, pixel sequences from scanned art work, word sequences from prose or property, textural sequences requiring some intrinsic variation, and so on.

(x) Other Areas

In chemistry, predicting gas solubility is essential to manufacturing polymers, but models using particle swarm optimization (PSO) tend to converge to the wrong points. An improved version of PSO has been created by introducing chaos, which keeps the simulations from getting stuck. In celestial mechanics, especially when observing asteroids, applying chaos theory leads to better predictions about when these objects will approach Earth and other planets. Four of the five moons of Pluto rotate chaotically.

Researchers have continued to apply chaos theory to psychology. For example, in modeling group behavior in which heterogeneous members may behave as if sharing to different degrees what in Wilfred Bion theory is a basic assumption, researchers have found that the group dynamic is the result of the individual dynamics of the members: each individual reproduces the group dynamics in a different scale, and the chaotic behavior of the group is reflected in each member.

Traffic forecasting may benefit from applications of chaos theory. Better predictions of when traffic will occur would allow measures to be taken to disperse it before it would have occurred. Combining chaos theory principles with a few other methods has led to a more accurate short-term prediction model (see the plot of the BML traffic model at right).

Chaos theory has been applied to environmental water cycle data (aka hydrological data), such as rainfall and stream flow. These studies have yielded controversial results, because the methods for detecting a chaotic signature are often relatively subjective. Early studies tended to "succeed" in finding chaos, whereas subsequent studies and meta-analyses called those studies into question and provided explanations for why these datasets are not likely to have low-dimension chaotic dynamics.

Limitations of the Chaos Theory

In this section we first will argue that limitations need to be acknowledged simply as part of human experience and that if properly conceptualized, they will assist both career counselors and their clients, to a deeper appreciation of reality and to more effective ways of successfully negotiating it. Next the nature of limitation will be examined and its implications for how we ought to think about our lives and careers.

The two major issues here and the limitation imposed by relativity (the fact that information cannot propagate faster than light) and the uncertainty principle. The first means that it is impossible within our laws of physics to simultaneously know everything, because the information must cross the distances at a relatively slow rate. The second is a fundamental principle of quantum mechanics you can never know the position and spin of a sub atomic particle. This is not, I repeat not, a human limitation. This is a fundamental property of the system the more precise you know one the more you lose the other. Chaos theory works similarly to statistics. You can't predict a certain behavior from a statistic but you can tell what range it should fall in. Same thing with chaos, an asteroids orbit may not pass through the exact point in a “perpendicular” plane, yet it may pass through the same region of that plain enough to make over all predictions about the orbit of the asteroid.

Chaos theory in itself sort of explains the difficulty involved in predicting the future to any degree of accuracy. Take weather for example. Weather patterns are a perfect example of Chaos Theory. We can usually predict weather patterns pretty well when they are in the near future, but as time goes on, more factors influence the weather, and it becomes practically impossible to predict what will happen. That example is analogous to most other Chaos Theory examples in that time is a huge limitation. As more time passes, more and more factors influence what can happen.

Control of Chaos

Control of chaos is the stabilization, by means of small system perturbations, of one of these unstable periodic orbits. The result is to render an otherwise chaotic motion more stable and predictable, which is often an advantage. The perturbation must be tiny compared to the overall size of the attractor of the system to avoid significant modification of the system's natural dynamics.

Several techniques have been devised for chaos control, but most are developments of two basic approaches: the OGY (Ott, Grebogi and Yorke) method, and Pyragas continuous control. Both methods require a previous determination of the unstable periodic orbits of the chaotic system before the controlling algorithm can be designed.

(i) OGY Method

E. Ott, C. Grebogi and J. A. Yorke were the first to make the key observation that the infinite number of unstable periodic orbits typically embedded in a chaotic attractor could be taken advantage of for the purpose of achieving control by means of applying only very small perturbations. After making this general point, they illustrated it with a specific method, since called the OGY method (Ott, Grebogi and Yorke) of achieving stabilization of a chosen unstable periodic orbit. In the OGY method, small, wisely chosen, kicks are applied to the system once per cycle, to maintain it near the desired unstable periodic orbit (Fradkov and Pogromsky 1998). The weaknesses of this method are in isolating the Poincaré section and in calculating the precise perturbations necessary to attain stability.

(ii) Pyragas Method

In the Pyragas method of stabilizing a periodic orbit, an appropriate continuous controlling signal is injected into the system, whose intensity is practically zero as the system evolves close to the desired periodic orbit but increases when it drifts away from the desired orbit. Both the Pyragas and OGY methods are part of a general class of methods called “closed loop” or “feedback’ methods which can be applied based on knowledge of the system obtained through solely observing the behavior of the system as a whole over a suitable period of time.

Experimental control of chaos by one or both of these methods has been achieved in a variety of systems, including turbulent fluids, oscillating chemical reactions, magneto-mechanical oscillators, and cardiac tissues (Schiff, S. J., Jerger, K. and Duong, D. H., Chang, T., Spano, M. L. and Ditto, W. L. 1994) attempt the control of chaotic bubbling with the OGY method and using electrostatic potential as the primary control variable.

Now we discuss the idea of controlling of chaos. There are three ways to control chaos (Garfinkelet, 1992):

(i) Alter organizational parameters so that the range of fluctuations is limited.

(ii) Apply small perturbations to the chaotic system to try and cause it to organize.

(iii)Change the relationship between the organization and the environment.

For many years, this feature made chaos undesirable, and most experimentalists considered such characteristic as something to be strongly avoided. Besides this critical sensitivity to initial conditions, chaotic systems exhibit two other important properties:

(a) There are an infinite number of unstable periodic orbits embedded in the underlying chaotic set. In other words, the skeleton of a chaotic attractor is a collection of an infinite number of periosic orbits, each one being unstable.

(b) The dynamics in the chaotic attractor is ergodic, which implies that during its temporal evolution the system ergodically visits small neighborhood of every point in each one of the unstable periodic orbits embedded within the chaotic attractor.

The above properties is that a chaotic dynamics can be seen as shadowing some periodic behavior at a given time, and erratically jumping from one to another periodic orbit. Indeed, if it is true that a small perturbation can give rise to a very large response in the course of time, it is also true that a judicious choice of such a perturbation can direct the trajectory to whenever one wants in the attractor, and to produce a series of desired dynamical states. This is the idea of control of chaos.

Conclusion

Chaos theory is a new way of thinking about what we have. It gives us a new concept of measurements and scales. It looks at the universe in an entirely different way. Understanding chaos understands life as we know it. Because of chaos, it is realized that even simple systems may give rise to and, hence, be used as models for complex behavior. Chaos forms a bridge between different fields. Chaos offers a fresh way to proceed with observational data, especially those data which may be ignored because they proved too erratic.

The specificity of present time physics, with entropy, chaos, and fractal dimensions, confers reality to phenomena as we can perceive and measure them, and it somehow invalidates the idea of a fundamental, or true, reality that might be explained by an elegant model. The use of such models entails too many simplifications, and may lead for instance to the reversibility of time that is imposed by the mathematical structure of mechanics.

More research should be conducted on that field in order to find out how to control the chaotic behavior of different systems in order to increase the validity of our future models and plans especially in the field of economics, where future models and plans can give crucial information about the general state of the economy in different countries.

In this paper, some basic applications of chaotic systems have been explored. Recently extensive research works in various other fields of mathematics have been going only to better understand these chaotic systems. In the last section of this paper we discuss the limitations of chaos theory and also establish the idea of control of chaos.

References

Baker, G. L and Gollub, J. P., 1990. Chaotic Dynamics: An Introduction, Cambridge University Press, Cambridge.

Biswas, H. R., 2014. One Dimensional Chaotic Dynamical Systems, J. P. App. Math.: Adv. and Appl. **12**: 1-24.

Blesser, B., 2006. Chaos Theory: Limits of Analysis and Predictions.

Boon, J. and Decroly, O. 1995. Dynamical Systems for Music Dynamics, chaos 5: 501-508.

Chen, G. and Ueta, T., 2002. Chaos in Circuits and Systems, World Scientific Publishing Co., Singapore, ISBN 981-02-4933-00.

Devaney, R. L and Keen, L., 1989. Chaos and Fractals: The Mathematics behind the Computer graphics, American Mathematical Society, Providence.

Ditto, W. and Pecora, L. 1993. Mastering Chaos in Scientific American, 78-84.

Fradkov A. L. and Pogromsky A.Yu. 1998. Introduction to Control of Oscillations and Chaos.Singapore: World Scientific Publishers.

Falconer, K., 1985. *The Geometry of Fractal Sets. Cambridge University Press.*

Falconer, K., 1997. *Techniques in Fractal Geometry. John Wiley.*

Falconer, K., 2014. *Fractal Geometry: Mathematical Foundations and Applications (3rd ed.). John Wiley & Sons.*

Goldbeter A. 2002.Computational approaches to cellular rhythms. *Nature.* 420:238–245.

Garfinkel, A. Spano, M. L., Ditto, W. L. and Weiss, J. 1992. Controlling Cardiac Chaos in Sciences 257: 1230-5.

Gierer A., and Meinhardt H. 1972. A theory of biological pattern. Kybernetik. 12:30–39.

Kiel and Elliott 1996. Chaos Theory in the Social Sciences: Foundations and Applications, Ann Arbor: The University of Michigan Press.

Lorenz, E. 1963. Deterministic Non periodic flow, Journal of Atmospheric Sciences.

Lorenz, E. 1993. The Essence of Chaos, London: *UCL Press*.

Morse, M. 1967. Rage for Chaos: Biology, Behavior and the Arts, New York: Schocken Books.

Oestreicher, C. 2007. A history of chaos theory. *Dialogues in Clinical Neurosciene, 9*:279-289.

Robert M. 1976. Simple Mathematical Models with very Complicated Dynamics, Nature 261, 459- 467.

Schiff, S. J., Jerger, K. and Duong, D. H., Chang, T., Spano, M. L. and Ditto, W. L. 1994. "Controlling Chaos in the Brain" in Nature, 370:615-20.

Townsend, J. 1992.Chaos Theory: A Brief Tutorial and Discussion, Indiana University.

Yasser, KRA. 2007. Chaos Theory and Literature from an Existentialist Perspective. CLC Web: Comparative Literature and Culture. <https://doi.org/10.7771/1481-4374.1226>

Zeng, X. 1993.Chaos theory and its application in the Atmosphere, Department of Atmospheric Science, Colorado States University, paper no. 504.